

University of Groningen

Interconnection of systems

Maschke, B.M.; Schaft, A.J. van der

Published in:
Proceedings of the 35th IEEE Conference on Decision and Control, 1996

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
1996

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Maschke, B. M., & Schaft, A. J. V. D. (1996). Interconnection of systems: the network paradigm. In *Proceedings of the 35th IEEE Conference on Decision and Control, 1996* University of Groningen, Johann Bernoulli Institute for Mathematics and Computer Science.

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Interconnection of systems: the network paradigm

B.M. Maschke

Laboratoire d'Automatisme Industriel, Conservatoire National des Arts et Métiers
Paris, FRANCE

e-mail: Bernhard.Maschke@paris.ensam.fr

A.J van der Schaft

Department of Applied Mathematics, University of Twente
Enschede, the NETHERLANDS

Abstract. *In this paper we propose first to recall the different interconnection structures appearing in network models and to show their exact correspondence with Dirac structures. This definition of interconnection is purely implicit hence does not discriminate between inputs and outputs among the interconnection variables and describes the relations between in an implicit form as some geometric space. Secondly we extend the definition of network interconnection by considering the variables defining the energy as belonging to Lie groups and will show that this leads to consider interconnection as Dirac structure on some Lie algebras.*

1. Introduction

The control theory considers the interconnection of systems as essentially external to the system, for instance in considering feedback interconnection. In general the connections considered consists is simple identities of the external variables. This holds also for the analysis of passive systems where the passivity preserving connection between systems consists also in the identity of external variables [29]. However concerning passive systems there is a broad class of models where the interconnection lies at the heart of the model: the network models as for instance electrical circuits, one- or three-dimensional mechanical systems and more generally analogue circuit models and bond graph models [12] [6] [21] [24]. Indeed in constructing a network model one proceeds in two steps: first tearing or reticulating the system into elemental components representing some physical phenomena, then interconnect the components through an interconnection network to a whole system [21] [11] [6] [2] [20].

It appeared very soon that the interconnection structure in network models actually defines on the system's level a geometric structure [12] [6] [24]. In a previous paper we have proposed to associate with the interconnection network a geometric structure called Dirac structure which encompasses the previously proposed geometric structures [15] [14] [27] and allows to deal in an intrinsic way with constrained systems or networks containing excess elements [27].

In this paper we propose first to recall the different interconnection structures appearing in network models and to show their exact correspondence with Dirac structures [3] [5] in order to end up with a purely geometric definition of interconnection of dynamical physical systems. This defi-

nition of interconnection has the advantage of being purely implicit hence it does not discriminate between inputs and outputs among the interconnection variables [29] and secondly it describes the relations between the interconnection variables not as maps but in an implicit form as some geometric space. In a second part we propose to extend the definition of network interconnection by considering the variables defining the energy as belonging to Lie groups and will show that this leads also to consider interconnection as defined by a Dirac structure, now on some Lie algebras.

2. Interconnection in networks with scalar variables

2.1. Internal and external variables

The first step in defining a network model of a physical system consists in tearing or reticulating a number of phenomena occurring in the system such as accumulation of elastic energy. Each phenomenon is then represented by a distinguished (multi-)port element among a finite number of types [21] [2]. The second step is then to define the whole system by interconnecting the different port elements through a network which in the simplest case may be an oriented graph, but may be of much more involved nature.

However in network models of physical systems the constitutive phenomenon is the storage of some kind of energy and consequently the definition of the associated (multi-)port element. The storage of energy is defined by a real variable $x \in \mathbb{R}$ or an real vector $x \in \mathbb{R}^n$, representing the energetic state of the system in the particular physical domains and called *energy variable*, and a real valued function $H(x)$ of the energy variable, called *energy function*. For instance in electrical circuit, the electrical energy, respectively the magnetic energy, are functions of the charge of the capacitors, respectively of the flux linkage of the inductors. Or the internal energy of a simple thermodynamical system is a function of its number of moles, its entropy and its volume. The energetic state in some physical domains of the system may undergo some changes in time which may be described using two variables, considered as external (or port) variables to the elemental energy storing system and called *power variables* following the bond graph terminology [21] [11], and which are defined as follows:

$$\left. \begin{aligned} f &= \frac{dx}{dt} \\ e &= dH(x) \end{aligned} \right\} \quad (1).$$

These variables are called *conjugated* in the sense that their duality product is the time variation of the energy function, i.e. represents the power flow ingoing the energy storage multiport:

$$\left\langle \frac{\partial H}{\partial x}, \frac{dx}{dt} \right\rangle = \frac{dH}{dt} \quad (2).$$

These power variables describing the interaction of the elemental energy storing system with the rest of the physical system, will also be the variables on which the interconnection network is defined. This interconnection network is an essential element of the model and indeed may itself also be considered and represented as a multiport element [21].

2.2. Interconnection in network models and their geometric structure

In this paragraph we shall recall the definition of interconnection in network representations of physical systems and show that it corresponds actually to a unique geometric structure: the Dirac structure [3] [5]. Indeed the interaction network is primarily power continuous, in other words it does not store energy, that means the power balance at its ports is equal to zero. This essential property should be embedded in the proposed geometric structure. A second point is that the interconnection network represents a finite number of physical phenomena and is hence endowed with very precise structures which we shall recall using a classification into intra- and inter- domain couplings. These interconnection structures should also be all be embedded in the proposed geometric structure. Before detailing the different interaction networks and showing their correspondence with Dirac structures, we shall first recall the definition of the Dirac structure.

A **Dirac structure** is defined on a vector space in an intrinsic geometric way as a subspace of the product of this space with its dual and it may be shown [27] that the original definition [3] is equivalent to the following one.

Definition 1: [27] Consider a n -dimensional vector space \mathcal{V}^n . A **Dirac structure** on \mathcal{V}^n is an n -dimensional subspace of $L \subset \mathcal{V}^n \times (\mathcal{V}^n)^*$ with the property that:

$$\forall (x, y) \in L, \langle x, y \rangle = 0 \quad (3).$$

This definition may immediately be commented with respect to the power continuity of interconnection network; considering the duality product as corresponding to the balance of power, it is seen that the Dirac structure embeds the concept of power continuity.

In order to provide a constructive definition of a Dirac structure, this intrinsic geometric definition of Dirac structures may be reformulated by use of linear maps as follows.

Proposition 1 :

(i) Every Dirac structure $L \subset \mathcal{V}^n \times (\mathcal{V}^n)^*$ may be written as: $L = \ker(F + E)$ for certain linear maps: $F: \mathcal{V}^n \rightarrow \mathcal{W}^n$ and $E: \mathcal{V}^{n*} \rightarrow \mathcal{W}^n$. Furthermore any such maps satisfy:

$$E^* F^* + F E^* = 0 \quad (4)$$

(ii) Every n -dimensional subspace defined by linear maps: $F: \mathcal{V}^n \rightarrow \mathcal{W}^n$ and $E: \mathcal{V}^{n*} \rightarrow \mathcal{W}^n$ according to (4) defines a Dirac structure.

A Dirac structure may also be defined on a differentiable manifold by considering as vector fields each fiber of its tangent and cotangent bundles.

Definition 2: [3] [5] A *generalized Dirac structure* on the differentiable manifold \mathcal{X} is given by a smooth vector sub-

bundle $L \subset T\mathcal{X} \oplus T^*\mathcal{X}$ such that the linear space: $L(x) \subset T_x\mathcal{X} \times T_x^*\mathcal{X}$ is a Dirac structure on $T_x\mathcal{X}$, for every $x \in \mathcal{X}$.

Note that we call the Dirac structure generalized as we do not require that it is integrable (see [27] [28]).

Intradomain coupling network. It is of a purely combinatorial nature and is called *network graph* or *port interconnection graph*. The most current class is defined by an oriented graph as it appears in electrical circuits, hydraulic circuits or in mechanical systems (one-dimensional as well as three-dimensional). With each edge of the graph a pair of conjugated power variables is associated. These variables are classed into either across- or through variables. Usually this classification is chosen according to meaning given to the nodes in the graph. For instance for electrical circuits a voltage is associated with any node and hence the voltages are the across variables and the currents are the through variables. For 1-dimensional mechanical systems, displacements or velocities are associated with every node and the across variables are then the velocities and the through variables are the forces. But one could as well choose the opposite classification, on the expense of using the dual port connection network. The network graph describes the connection constraints among the port variables of the elements due to Kirchhoff's laws which may be formulated in the following generalized form.

Proposition 2 Kirchhoff's loop law:

The sum of across variables along any cycle (or loop) in the network graph vanishes.

Proposition 3 Kirchhoff's cutset law:

The sum of through variables along any cocycle (or cutset) in the network graph vanishes.

It is immediate that the two Kirchhoff's laws actually defines a Dirac structure on the space of through and across variables by Tellegen's theorem.

Proposition 4 Tellegen's theorem:

Let \mathcal{G} be a port connection graph and denote the through variables by $i \in \mathbb{R}^n$ and the across variables by $v \in \mathbb{R}^n$, the latter being considered to lie in the dual real space to the through variables, then Kirchhoff's laws implies:

$$\langle v, i \rangle = 0 \quad (5).$$

The Dirac structure may also be defined in a constructive way by using the fundamental loop matrix associated with any maximal tree in the graph. Denote by \mathcal{T} a maximal tree of the network graph \mathcal{G} and by $\overline{\mathcal{T}}$ the cotree complementing \mathcal{T} in the network graph \mathcal{G} . Then Kirchhoff's laws imply the two following relations on the port variables of the elements:

$$(I_{\overline{\mathcal{T}} \times \overline{\mathcal{T}}} B_{\mathcal{T}}) \begin{pmatrix} v_{\overline{\mathcal{T}}} \\ v_{\mathcal{T}} \end{pmatrix} = 0 \quad (6)$$

$$(-B'_{\mathcal{T}} I_{\mathcal{T} \times \mathcal{T}}) \begin{pmatrix} i_{\overline{\mathcal{T}}} \\ i_{\mathcal{T}} \end{pmatrix} = 0 \quad (7)$$

where $B = (I_{\overline{\mathcal{T}} \times \overline{\mathcal{T}}} B_{\mathcal{T}})$ is the fundamental loop matrix, and $Q = (-B'_{\mathcal{T}} I_{\mathcal{T} \times \mathcal{T}})$ is the fundamental cutset matrix associated with the tree \mathcal{T} , $v_{\mathcal{T}}$, $v_{\overline{\mathcal{T}}}$ (respectively $i_{\mathcal{T}}$, $i_{\overline{\mathcal{T}}}$) denotes the across variables (respectively the through variables) associated with the edges in \mathcal{T} and $\overline{\mathcal{T}}$ and $B_{\mathcal{T}}$ is an $n_{\overline{\mathcal{T}}} \times n_{\mathcal{T}}$ matrix with coefficients in $\{-1, 0, 1\}$. The equations (6) and (7) may also be written as follows:

$$\begin{pmatrix} I_{\mathcal{T} \times \mathcal{T}} & B_{\mathcal{T}} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v_{\mathcal{T}} \\ v_{\mathcal{T}} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -B_{\mathcal{T}}' & I_{\mathcal{T} \times \mathcal{T}} \end{pmatrix} \begin{pmatrix} i_{\mathcal{T}} \\ i_{\mathcal{T}} \end{pmatrix} = 0 \quad (8).$$

It is then easy to see that this defines indeed a Dirac structure according to Proposition 1.

Interdomain coupling network. We shall first consider interdomain couplings as they occur in the elasto-kinetic coupling in one-dimensional mechanical systems or in the electro-magnetic coupling of electrical and magnetic fields in electrical circuits. As elementary such systems consider the 2-dimensional systems consisting of a mass-spring system or an LC-circuit. In the bond graph formalism these systems are modelled by two energy storing element coupled by a 2-port element called *symplectic gyrator* [1] defined by the constitutive relation:

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \quad (9)$$

where (e_i, f_i) are the power variable of the ports $i = 1, 2$. It is clear that the constitutive relation of the symplectic gyrator defines a Dirac structure on the conjugated power variables. It may be noted that the Dirac structure corresponds to the definition of a symplectic Poisson bracket on the manifold of the energy variables [15] and that the dynamic systems are standard hamiltonian systems.

The interdomain coupling may also be dependent on the energy variables, for instance for interdomain coupling with the thermal domain, for instance in thermo-chemical systems [8] or thermic systems [17] and is then expressed in bond graph terms by a 2-port element called *irreversible transducer* [2] whose constitutive relation is analogous to the one of the gyrator and defines a non constant Dirac structure on the differential manifold of the energy variables. Another case where the interdomain coupling may depend on the energy variables is the case where the energy variables have the structure of a Lie group; this case will be treated in more details in section 3.

Finally the two-port interdomain coupling may be generalized to multiport interdomain coupling by coupling a combinatorial intradomain coupling network with 2-port interdomain coupling elements. This may be illustrated on the example of LC circuits (and by analogy on 1-dimensional mechanical systems or hydraulic systems).

Consider an LC-circuit (with possibly elements in excess) composed of a set of n_L inductors and n_C capacitors interconnected through the port connection graph \mathcal{G} . Choose a maximal tree \mathcal{T} in \mathcal{G} which is maximal with respect to the number n_{C_1} of capacitors connected to its edges and denote by $\overline{\mathcal{T}}$ its complementary cotree (which is maximal with respect to the number n_{L_1} of inductors connected to its edges). Denote furthermore by \mathcal{C}_1 (respectively \mathcal{C}_2) the maximal partial graph in \mathcal{T} whose edges correspond to capacitors' ports, which necessarily forms a tree (respectively the partial graph containing all capacitors in $\overline{\mathcal{T}}$) and by \mathcal{L}_1 (respectively \mathcal{L}_2) the maximal partial graph in $\overline{\mathcal{T}}$ whose edges correspond to the inductors' ports, which necessarily forms a cotree (respectively the partial graph containing all inductors in \mathcal{T}). Hence:

$$\mathcal{C}_1 \cup \mathcal{L}_2 = \mathcal{T} \text{ and } \mathcal{L}_1 \cup \mathcal{C}_2 = \overline{\mathcal{T}} \quad (10).$$

According to the partition of the tree \mathcal{T} and the cotree $\overline{\mathcal{T}}$

and the partition of the voltages and currents:

$$v' = \begin{pmatrix} v_{L_1}' & v_{C_2}' & v_{C_1}' & v_{L_2}' \end{pmatrix} \quad (11)$$

$$i' = \begin{pmatrix} i_{L_1}' & i_{C_2}' & i_{C_1}' & i_{L_2}' \end{pmatrix} \quad (12),$$

the fundamental loop matrix becomes:

$$B = \begin{pmatrix} I_{n_{L_1}} & 0_{n_{L_1} \times n_{C_2}} & B_{11} & B_{12} \\ 0_{n_{C_2} \times n_{L_1}} & I_{n_{C_2}} & B_{21} & 0_{n_{C_2} \times n_{L_2}} \end{pmatrix} \quad (13)$$

and the associated fundamental cutset matrix becomes:

$$Q = \begin{pmatrix} -B_{11}' & -B_{21}' & I_{n_{C_1}} & 0_{n_{C_1} \times n_{L_2}} \\ -B_{12}' & 0_{n_{L_2} \times n_{C_2}} & 0_{n_{L_2} \times n_{C_1}} & I_{n_{L_2}} \end{pmatrix} \quad (14).$$

As now the port connection graph connects two different types of elements: the capacitors and the inductors, it defines actually an interdomain coupling. This may be represented in network terms, according to the bond graph formalism, by augmenting the port connection graph with symplectic gyrators at the edges where the inductors are connected. Hence the choice of the vector space, on which the Dirac structure is defined, is modified according to the manifold of the energy variables: the charges of the capacitors and the flux linkages of the inductors. Indeed the port variables of the port connection graph are also port variables of the capacitors and inductors. Hence the currents at the ports of the capacitors and the voltages at the ports of the inductors constitute the tangent space to the manifold of energy variables and the set of conjugated variables constitute the cotangent space. Now from Kirchhoff's laws and the decomposition of the fundamental loop and cutset matrices, one obtains the following relation: (15)

$$\begin{pmatrix} I_{n_{C_1}} & 0 & -B_{21}' & 0 \\ 0 & I_{n_{L_1}} & 0 & B_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} i_{e_1} \\ v_{L_1}' \\ i_{e_2} \\ v_{L_2}' \end{pmatrix} = \begin{pmatrix} 0 & B_{11} & 0 & 0 \\ -B_{11} & 0 & 0 & 0 \\ B_{21} & 0 & I_{n_{C_2}} & 0 \\ 0 & -B_{12}' & 0 & I_{n_{L_2}} \end{pmatrix} \begin{pmatrix} v_{e_1} \\ i_{L_1}' \\ v_{e_2} \\ i_{L_2}' \end{pmatrix}$$

Again it is easy to verify that this relation defines a Dirac structure in the sense of Proposition 1.

2.3. Hamiltonian systems defined on Dirac structures and the generalization of interconnection

In this paragraph we shall firstly briefly recall the definition of an implicit hamiltonian system with ports defined on a Dirac structure [27] and secondly use these systems in order to extend the interdomain coupling.

Implicit hamiltonian system with ports [27]. The implicit hamiltonian systems with ports generalize the hamiltonian systems with ports [14] [27] in the sense that they include also hamiltonian systems with constraints or associated with energy conserving network models with elements in excess [27]. Let \mathcal{H} denote the space of the energy variables of a physical system. The space of port variables is then defined by $W_{\mathcal{H}}$ a vectorbundle over \mathcal{H} with the m -dimensional fiber $W_{\mathcal{H}}(x)$ defining the space of port flow variables. The conjugated effort variables then belong to $W_{\mathcal{H}}^*$. Denote by $T\mathcal{H} \oplus W_{\mathcal{H}} \oplus T^*\mathcal{H} \oplus W_{\mathcal{H}}^*$ the vectorbundle over \mathcal{H} with fibers: $T_x\mathcal{H} \times W_{\mathcal{H}}(x) \times T_x^*\mathcal{H} \times W_{\mathcal{H}}^*(x)$. An implicit hamiltonian system is the defined in the following way, where the geometric structure on the space of energy variables and port variables is defined by a Dirac structure, playing an

analogous role to the Poisson bracket in explicit hamiltonian systems.

Definition 3: An implicit hamiltonian system with ports is defined by a space of energy variable \mathfrak{G} , vectorbundles W_f and W_f^* over \mathfrak{G} and a smooth vectorbundle $L \subset T\mathfrak{G} \oplus W_f \oplus T^*\mathfrak{G} \oplus W_f^*$ such that $L(x) \subset T_x\mathfrak{G} \times W_f(x) \times T_x^*\mathfrak{G} \times W_f^*(x)$ is a linear Dirac structure over $T_x\mathfrak{G} \times W_f(x)$ for every $x \in \mathfrak{G}$. Here $(T_x\mathfrak{G} \times W_f(x))^*$ is identified with $T_x^*\mathfrak{G} \times W_f^*(x)$ by using the natural identification:

$$(e, w_e) \in T_x^*\mathfrak{G} \times W_f^*(x) \mapsto \beta = (e, -w_e) \in (T_x\mathfrak{G} \times W_f(x))^* \quad (16)$$

with e and $-w_e$ the components of β .

The implicit hamiltonian system with respect to the hamiltonian function $H: \mathfrak{G} \rightarrow \mathbb{R}$ is given defining that for all $x \in \mathfrak{G}$:

$$(f(x) = \dot{x}(x), w_f(x), e(x) = dH(x), w_e(x)) \in L(x) \quad (17)$$

with $\dot{x} \in T_x\mathfrak{G}$, the velocity of the system in state x .

An important class of such models consists in energy storing n -port elements with constitutive relation (1). These constitutive relations may be written in terms of a implicit hamiltonian system with ports as follows:

$$\begin{pmatrix} I & -I \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{x} \\ f \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ I & I \end{pmatrix} \begin{pmatrix} dH \\ -e \end{pmatrix} = 0 \quad (18).$$

A second example is the LC circuits treated above. Identifying now the currents and voltages at the ports of the interconnection network with port variables of the capacitors and inductors of the circuit and denoting by q_e the vector of charges of the capacitors and ϕ_e the vector of the flux linkages of the inductors and by E_C the total electrical energy and by E_L the total magnetic energy, the equations (15) becomes:

$$\begin{pmatrix} I_{nC_1} & 0 & -B_{21}^t & 0 \\ 0 & I_{nL_1} & 0 & B_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{\phi}_1 \\ \dot{q}_2 \\ \dot{\phi}_2 \end{pmatrix} = \begin{pmatrix} 0 & B_{11}^t & 0 & 0 \\ -B_{11} & 0 & 0 & 0 \\ B_{21} & 0 & I_{nC_2} & 0 \\ 0 & -B_{12}^t & 0 & I_{nL_2} \end{pmatrix} \begin{pmatrix} \frac{\partial E_C}{\partial q_1} \\ \frac{\partial E_L}{\partial \phi_1} \\ \frac{\partial E_C}{\partial q_2} \\ \frac{\partial E_L}{\partial \phi_2} \end{pmatrix}.$$

This defines an implicit hamiltonian system (without ports) with respect to the Dirac structure defined in (15) and with respect to the total electromagnetic energy of the circuit.

Interdomain coupling through hamiltonian systems. Implicit hamiltonian system with ports allow to encompass other interdomain couplings essential to actuator and sensor models: the interdomain coupling through an energy function, in network terms through an n -port energy storing element. Hence the structure of the coupling is now defined not only by a Dirac structure (in this case the trivial structure in equation (18)), but also an energy function and the associated hamiltonian system (18).

Consider for example a simplified model of a magnetic suspension consisting of an iron ball in 1-dimensional displacement in the magnetic field generated by an electromagnet. Disregarding first the kinetic energy of the ball, this system may be modeled as an 2-port element storing magnetic energy H_{mg} . The energy variables are the flux linkage ϕ and the position z of the ball and the energy may

$$\text{be expressed as follows: } H_{mg}(\phi, z) = \frac{1}{2} \frac{\phi^2}{L(z)} \quad (20)$$

where $L(z)$ is the inductance which depends on the displacement z . This system couples the electromagnetic energy supply with the mechanical suspended system and, as a 2-port energy storing element, is a hamiltonian system. Finally this definition of interdomain coupling may obviously be generalized to coupling through any implicit hamiltonian systems with ports.

This may again be illustrated with the example of a magnetic suspension, where the kinetic energy of the ball :

$$H_{kin}(p) = \frac{1}{2} \frac{p^2}{m} \quad (21)$$

with mass m and momentum p would be taken into account. The suspension is the following hamiltonian system with ports:

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{z} \\ \dot{p} \\ f_{mg} \\ f_{mec} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\partial H_{mg}}{\partial \phi} \\ \frac{\partial H_{mg}}{\partial z} \\ \frac{\partial H_{kin}}{\partial p} \\ -e_{mg} \\ -e_{mec} \end{pmatrix} \quad (22)$$

where (f_{mg}, e_{mg}) are the port variables at the electromagnetic port and (f_{mec}, e_{mec}) are the port variables at the mechanical port.

3. Interconnection in networks defined on Lie groups

In the previous section we have considered energy variables being reals or real vectors. However this definition obscures very much the models and particularly the interconnection structure of these models when considering systems as multibody systems, where the displacements may not be expressed as real vectors but belong intrinsically to the Lie group of rigid body displacements. In the sequel we propose to use the Lie group structure in order to generalize the definition of interconnection to any Lie group on which the energy variable may be defined, based on previous work on multibody systems [17] [19].

3.1. Energy variables as elements of a Lie group

Consider now an energy storing phenomenon where the energy variable belong to some Lie group G [13], for instance the group of rigid body displacements $SE(3)$. The definition of the energy storing element then remains analogue but the definition of the port variables as defined in (1) leads to some problem. Indeed these port variables are then defined in the tangent and cotangent space at the given configuration of the energy and hence are local to the element and not suitable as port variables of the interconnection network. However using the Lie group structure of the energy variables leads to define naturally as port variables the elements of the Lie algebra \mathfrak{g} and its dual \mathfrak{g}^* , associated with the Lie group G [13]. Extending the work on mechanical networks in [17] [19], the port variables are then defined as follows with respect to the velocity and the differential of the energy:

$$f = TR_{Q^{-1}} \left(\frac{d}{dt} Q \right) \quad (23)$$

$$e = T^*R_Q(dH) \quad (24)$$

where TR_Q denotes the tangent map to the right translation

at Q [13].

It is clear that these port variables on the one hand do not depend on the configuration Q and on the other hand allow to compute the port variables derived from the energy function in equations (1).

In the case of G being the group of rigid body displacement, the variable f are the velocity in fixed frame, called *twists*, in the Lie algebra $se(3)$ and e is the force in fixed frame, called *wrench*, in the dual Lie algebra $se^*(3)$.

The port variables of the energy storing elements will, as in the scalar case, be used for the definition of the interconnection network.

3.2. Interconnection in Lie-network models

Intradomain coupling. Defining as port variables the elements of the Lie algebra \mathfrak{g} and its dual \mathfrak{g}^* and using the vector space structure of these spaces, the combinatorial interconnection network may easily be extended. For instance it may be defined by using oriented graphs and the elements of the Lie algebra and its dual as the across and through variables. By applying Kirchhoff's cycle and cocycle laws on the network graph on defines a Dirac structure on the product Lie algebra \mathfrak{g}^n where n is the number of edges of the graph. In the case of multibody systems, the interconnection graph is readily defined from the topological structure of the mechanism and it may be shown that Kirchhoff's laws applies on the twists and wrenches in fixed frame [4].

Furthermore, as the Lie algebras may be of dimension higher than one, some additional intradomain coupling may occur. Actually in general they may simply be defined as a Dirac structure defined on the Lie algebra \mathfrak{g} associated with the Lie group G where the energy variable lies. In network formalism these coupling may be represented by an 1-port element. Considering multibody systems, such elements represent the kinematic pairs interconnecting the bodies [17] [19].

Interdomain coupling. The interdomain coupling has a more complex structure then in the case of scalar energy variables, which may be deduced from the Lie group structure of the space of energy variables. Consider firstly the interdomain coupling between the kinetic and the potential energy in the rigid body dynamics, that is when the Lie group of displacements is the group of positively oriented isometries in \mathbb{R}^3 .

Indeed the dynamics of a rigid body endowed with some potential energy $U(Q)$ depending on its position $Q \in SE(3)$, and kinetic energy $K(P) = \frac{1}{2} \langle P, Y P \rangle$ where P is its momentum in body frame and Y is its mobility tensor (the inverse of the inertia tensor) [9] may be expressed by the following system [17] [19]:

$$\frac{d}{dt} \begin{pmatrix} Q \\ P \end{pmatrix} = \begin{pmatrix} 0 & TL_Q \\ -T^*L_Q & -P \times \end{pmatrix} \begin{pmatrix} dU(Q) \\ Y P \end{pmatrix} + \begin{pmatrix} 0 \\ Ad_Q^* \end{pmatrix} W_B$$

$$T_B = \begin{pmatrix} 0 & Ad_Q \end{pmatrix} \begin{pmatrix} 0 \\ Y P \end{pmatrix}$$

where \times denotes the Lie-Poisson bracket, TL_Q is the tangent map to the left translation, Ad_Q is the adjoint representation of $SE(3)$ on $se(3)$ [13]. The twist T_B is the velocity in fixed frame of the body and the wrench W_B in fixed frame represents the external forces applied to the rigid body.

This dynamical system may be considered as a hamiltonian system with ports. Indeed the space of energy variables consists in the pair $(Q, P) \in SE(3) \times se^*(3)$ of rigid body displacements and momentum in body frame. The tangent space, at some point of the manifold of energy variables, may then be identified with pairs $(v, T) \in T_Q SE(3) \times se(3)$ of velocities and twists in body frame. The port variables are pairs $(W_B, T_B) \in se^*(3) \times se(3)$ of wrenches and twists in fixed frame. Consider now the following Dirac structure defined on the vector space: $T_Q SE(3) \times se(3) \times se^*(3)$:

$$\begin{pmatrix} I & 0 & 0 \\ 0 & I & -Ad_Q^* \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v \\ T \\ W_B \end{pmatrix} + \begin{pmatrix} 0 & -TL_Q & 0 \\ T^*L_Q & P \times & 0 \\ 0 & Ad_Q & I \end{pmatrix} \begin{pmatrix} F \\ W \\ -T_B \end{pmatrix} = 0 \quad (26)$$

with elements denoted by:

$$(v, T, W_B) \in T_Q SE(3) \times se(3) \times se^*(3),$$

and with elements in its dual space denoted by:

$$(F, W, T_B) \in T^* SE(3) \times se^*(3) \times se(3).$$

Hence the dynamics of a rigid body described in equation (25), is a hamiltonian system with ports defined with respect to the Dirac structure (26) and with hamiltonian function equal to the sum of the potential and kinetic energies.

It may be noted that the Dirac structure defined in (26) in completely determined by the geometric structure of the Lie group where the displacement variables lie. Moreover two different network representations, in bond graphs, of this Dirac structure were given in [17] [19]. These representation make clearly appear the difference between the interdomain coupling in the scalar and the general Lie group which is represented by additional network elements. Finally this elementary interdomain coupling may be combined with the combinatorial interconnection and some Dirac structure defined on the Lie algebra, to construct complex interdomain interconnections, as it is for instance the case in multibody systems. And in some analogous way to the scalar case interdomain coupling may also be constructed through energy storing elements.

4. Conclusion

In this paper we have shown that the paradigm of network models gives a very precise definition of interconnection which is much broader than the usual interconnection between systems usually considered in control theory. We have shown that the interconnection may be defined as a geometric structure. Following the definition of network interconnection this geometric structure was shown correspond to a Dirac structure. We have furthermore given a precise definition of interconnection in the case when the variables defining the energetic state of the system, the energy variables, are no more scalars but belong to a Lie group.

The interconnection structures proposed in this paper, i.e. the Dirac structures, encompass various formulations of dynamic system as constraint systems and implicit systems. They have a very rich mathematical structure which is examined in details in a companion paper. It is hoped and further work will be devoted to this topic, that enlarging the definition of interconnection allows to develop some origi-

nal control synthesis procedure in particular in the control of systems interaction with their environment [9] [10].

5. References

- [1] P. C.Breedveld, "Thermodynamic bond graphs and the problem of thermal inertance", *J. of The Franklin Institute*, 314, 15-40, 1982
- [2] P.C.Breedveld, *Physical systems theory in terms of bond graphs* Ph.D. thesis, University of Twente, the Netherlands, 1984
- [3] T.J.Courant, "Dirac manifolds", *Trans. American Mathematical Society*, Vol. 319, n° 2, pp.631-661, June 1990
- [4] T. H. Davies, "Mechanical networks - I: Passivity and redundancy - II: Formulae for the degree of mobility and redundancy - III: Wrenches on circuit screws", *Mechanism and Machine Theory*, vol. 18 : 95-112 1983.
- [5] I.Dorfman, *Dirac Structures and Integrability of Nonlinear Evolution Equations*, John Wiley, Chichester, 1993
- [6] O.I.Franksen, "Kron's method of tearing", *Proc. 4th Power and Computer Appl. Conf.*, Clearwater, Folrida, pp.7-38, USA, 1965
- [7] O.I.Franksen, "Are data structures geometrical objects? I: Invoking the Erlanger Program", *Syst. Anal. Model. Simul.*, Vol.1, n°2, pp.113-130, 1984
- [8] E.P.Gyftopoulos and G.P.Beretta, "Entropy Generation rate in a Chemically Reacting System", preprint, 1992 *WAM of the ASME*, Anaheim, California, USA, 1992
- [9] N.Hogan, "Impedance Control : an Approach to Manipulation: Part 1 - Theory", *Trans. ASME, J. Dynamic Systems and Measurements*, vol. 107, No. 1, pp. 1-7, 1985
- [10] N.Hogan, "Beyond regulators: modelling control systems as physical systems", *Proc. of the 1987 American Control Conf.*, Minneapolis, MN, June 1987
- [11] D.C.Karnopp, D.L.Margolis and R.C.Rosenberg, *Systems Dynamics : A Unified Approach*, Second Edition, John Wiley & Sons, New-York, U.S.A, 1990
- [12] G.Kron, "Tearing and interconnection as a form of transformation", *RAAG Memoirs*, Vol.XIII, n°2, pp.147-159
- [13] Libermann P. and Marle C.M., *Symplectic geometry and Analytical Mechanics*, Reidel, Dordrecht, 1987
- [14] B.M. Maschke and A.J. van der Schaft, "Port controlled Hamiltonian systems: modeling origins and system theoretic properties", *Proc. 2nd IFAC Symp. on Nonlinear Control Systems Design*, NOLCOS'92, pp.282-288, Bordeaux, June 1992
- [15] B.M.Maschke, A.J.van der Schaft and P.C.Breedveld, "An Intrinsic Hamiltonian Formulation of Network Dynamics: Nonstandard Poisson Structures and Gyrators", *Journal of the Franklin Institute*, Vol. 329, n. 5, pp. 923-966, 1992
- [16] B.M. Maschke and P.Chantre, "Bond graph modeling and parameter identification of a heat exchanger", *Proc. of the 1994 ASME Int. Mechanical Engineering Congress and Exposition*, DSC-Vol. 55-2 pp.645-652, Chicago, USA, November 6-11 1994
- [17] B.M. Maschke, C.Bidard and A.J. van der Schaft, "Screw-vector bond graphs for the kinestatic and dynamic modeling of multibody systems", *Proc. of the 1994 ASME Int. Mechanical Engineering Congress and Exposition*, DSC-Vol. 55-2 pp.637-644, Chicago, USA, November 6-11 1994
- [18] B.M.Maschke, A.J. van der Schaft and P.C.Breedveld, "An intrinsic Hamiltonian formulation of the dynamics of LC-circuits", *Trans. IEEE on Circuits and Systems, I: Fundamental Theory and Applications*, Vol.42, n°2, pp.73-82, February 1995.
- [19] B.M.Maschke, "Elements on the Modelling of Multibody sytems" in *Modelling and Control of Mechanisms and Robots*, C.Melchiorri and A.Tornamè editors, World Scientific Publishing Ltd., 1996
- [20] Oster G. F. and Perelson A. S., "Chemical reaction dynamics - part I: Geometrical structure", *Arch. Rational Mech. Anal.*, 55, 230-273, 1974
- [21] H.M.Paynter, *Analysis and Design of Engineering Systems*, M.I.T. Press, Cambridge, MA, 1961
- [22] H.M.Paynter, "Multiport dissipators as ideal work-into-heat converters", in W.G.Vogt and M.H.Mickle eds., *Proc. of the 10th annual Pittsburg Conference*, pp. 1943-1845, Instruments Society of America, April 25-27, 1979
- [23] Perelson A. S., "Description of electrical networks using bond graphs", *Circuit Theory and Applications*, 4, 107-123, 1976
- [24] G. F. Perelson A. S. and Oster, "Chemical reaction dynamics - part II: Reaction networks", *Arch. Rational Mech. Anal.*, Vol. 57, n°3, 31-98, 1975
- [25] Perelson A. S. and G.F.Oster, "Bond graphs and linear graphs", *J. of the Franklin Institute*, 302, n°2, pp.159-185, 1976
- [26] A.J. van der Schaft, "System theory and mechanics", in *Three Decades of Mathematical System Theory*, H.Nijmeier and J.M.Schumacher eds., Lect. Notes Contr. Inf. Sci., 135, Springer, Berlin, 1989
- [27] A.J. van der Schaft and B.M.Maschke, "The hamiltonian formulation of energy conserving physical systems with ports", *Archiv für Elektronik und Übertragungstechnik*, Vol.49, n°5/6, pp.362-371, 1995.
- [28] A.J. van der Schaft, M.Dalsmo and B.M.Maschke, "Mathematical structures in the network representation of energy conserving physical systems", in this proceeding.
- [29] J.C.Willems, "Dissipative Dynamical Systems, Part 1: General Theory", *Arch. Rat. Mech. Anal.*, Vol.45, pp.321-351, 1972